The first customer's service time is 4 minutes, because the random digits 84 fall in the bracket 61–85—or, alternatively, because the derived random number 0.84 falls between the cumulative probabilities 0.61 and 0.85.

The essence of a manual simulation is the simulation table. These tables are designed for the problem at hand, with columns added to answer the questions posed. The simulation table for the single-channel queue, shown in Table 2.10, is an extension of the type of table already seen in Table 2.4. The first step is to initialize the table by filling in cells for the first customer. The first customer is assumed to arrive at time 0. Service begins immediately and finishes at time 4. The customer was in the system for 4 minutes. After the first customer, subsequent rows in the table are based on the random numbers for interarrival time, service time, and the completion time of the previous customer. For example, the second customer arrives at time 1. But service could not begin until time 4; the server (checkout person) was busy until that time. The second customer waited in the queue for three minutes. The second customer was in the system for 5 minutes. Skip down to the fifth customer. Service ends at time 16, but the sixth customer does not arrive until time 18, at which time service began. The server (checkout person) was idle for two minutes. This process continues for all 100 customers. The rightmost two columns have been added to collect statistical measures of performance, such as each customer's time in system and the server's idle time (if any) since the previous customer departed. In order to compute summary statistics, totals are formed as shown for service times, time customers spend in the system, idle time of the server, and time the customers wait in the queue.

Some of the findings from the simulation in Table 2.10 are as follows:

1. The average waiting time for a customer is 1.74 minutes. This is computed in the following manner:

$$\text{Average waiting time (minutes)} = \frac{\text{total time customers wait in queue (minutes)}}{\text{total numbers of customers}} = \frac{174}{100} = 1.74 \text{ minutes}$$

2. The probability that a customer has to wait in the queue is 0.46. This is computed in the following manner:

$$\text{Probability (wait)} = \frac{\text{numbers of customers who wait}}{\text{total number of customers}} = \frac{46}{100} = 0.46$$

3. The proportion of idle time of the server is 0.24. This is computed in the following manner:

$$\text{Probability of idle server} = \frac{\text{total idle time of server (minutes)}}{\text{total run time of simulation (minutes)}} = \frac{101}{418} = 0.24$$

The probability of the server's being busy is the complement of 0.24, namely, 0.76.

Table 2.10 Simulation Table for Single-Channel Queuing Problem

		Clock		*Clock*		*Clock*		
Customer	*Interarrival Time (Minutes)*	*Arrival Time*	*Service Time (Minutes)*	*Time Service Begins*	*Waiting Time in Queue (Minutes)*	*Time Service Ends*	*Time Customer Spends in System (Minutes)*	*Idle Time of Server (Minutes)*
1		0	4	0	0	4	4	
2	1	1	2	4	3	6	5	0
3	1	2	5	6	4	11	9	0
4	6	8	4	11	3	15	7	0
5	3	11	1	15	4	16	5	0
6	7	18	5	18	0	23	5	2
7	5	23	4	23	0	27	4	0
8	2	25	1	27	2	28	3	0
9	4	29	4	29	0	33	4	1
10	1	30	3	33	3	36	6	0
11	4	34	5	36	2	41	7	0
12	4	38	3	41	3	44	6	0
13	7	45	4	45	0	49	4	1
14	6	51	5	51	0	56	5	2
15	3	54	3	56	2	59	5	0
16	8	62	2	62	0	64	2	3
17	8	70	4	70	0	74	4	6
18	2	72	3	74	2	77	5	0
19	7	79	1	79	0	80	1	2
20	4	83	2	83	0	85	2	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	5	415	2	416	1	418	3	0
Total	415		317		174		491	101

4. The average service time is 3.17 minutes. This is computed in the following manner:

$$\begin{aligned}\text{Average service time (minutes)} &= \frac{\text{total service time (minutes)}}{\text{total number of customers}} \\ &= \frac{317}{100} = 3.17 \text{ minutes}\end{aligned}$$

This result can be compared with the expected service time by finding the mean of the service-time distribution, using the equation

$$E(S) = \sum_{s=0}^{\infty} sp(s)$$

Applying the expected-value equation to the distribution in Table 2.7 gives

$$\text{Expected service time} =$$
$$1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05) = 3.2 \text{ minutes}$$

The expected service time is slightly higher than the average service time in the simulation. The longer the simulation, the closer the average will be to $E(S)$.

5. The average time between arrivals is 4.19 minutes. This is computed in the following manner:

$$\begin{aligned}\text{Average time between arrivals (minutes)} &= \frac{\text{sum of all times between arrivals (minutes)}}{\text{number of arrivals} - 1} \\ &= \frac{415}{99} = 4.19 \text{ minutes}\end{aligned}$$

One is subtracted from the denominator because the first arrival is assumed to occur at time 0. This result can be compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are $a = 1$ and $b = 8$. The mean is given by

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ minutes}$$

The expected time between arrivals is slightly higher than the average. However, as the simulation becomes longer, the average value of the time between arrivals should approach the theoretical mean, $E(A)$.

6. The average waiting time of those who wait is 3.22 minutes. This is computed in the following manner:

$$\begin{aligned}\text{Average waiting time of those who wait (minutes)} &= \frac{\text{total time customers wait in queue (minutes)}}{\text{total number of customers that wait}} \\ &= \frac{174}{54} = 3.22 \text{ minutes}\end{aligned}$$

7. The average time a customer spends in the system is 4.91 minutes. This can be found in two ways. First, the computation can be achieved by the following relationship:

$$\begin{array}{c}\text{Average time customer}\\ \text{spends in the system}\\ \text{(minutes)}\end{array} = \frac{\text{total time customers spend in the system (minutes)}}{\text{total number of customers}}$$

$$= \frac{491}{100} = 4.91 \text{ minutes}$$

The second way of computing this same result is to realize that the following relationship must hold:

$$\begin{array}{c}\text{Average time}\\ \text{customer spends}\\ \text{in the system}\\ \text{(minutes)}\end{array} = \begin{array}{c}\text{average time}\\ \text{customer spends}\\ \text{waiting in the}\\ \text{queue (minutes)}\end{array} + \begin{array}{c}\text{average time}\\ \text{customer spends}\\ \text{in service}\\ \text{(minutes)}\end{array}$$

From findings 1 and 4, this results in

$$\text{Average time customer spends in the system} = 1.74 + 3.17 = 4.91 \text{ minutes}$$

A decision maker would be interested in results of this type, but a longer simulation would increase the accuracy of the findings. However, some tentative inferences can be drawn at this point. About half of the customers have to wait; however, the average waiting time is not excessive. The server does not have an undue amount of idle time. More reliable statements about the results would depend on balancing the cost of waiting against the cost of additional servers.

Excel spreadsheets have been constructed for each of the examples in this chapter. The spreadsheets can be found at www.bcnn.net. The spreadsheets have a common format. The first sheet is One-Trial. The second sheet is Experiment. The third sheet is entitled Explain. Here, the logic in the spreadsheet is discussed, and questions pertaining to that logic are asked of the reader. Use the default seed '12345' to reproduce the One-Trial output shown in the examples in the text, and use the appropriate number of trials (or replications) to reproduce the Experiment shown in the text, again using the default seed '12345'.

Exercises relating to the spreadsheets have been prepared also. These are the last set of exercises at the end of this chapter. The first set of exercises is for manual simulation.

The spreadsheets allow for many entities to flow through the system. (In Example 2.1, the entities are customers.) For instance, the spreadsheet for Example 2.1 has 100 customers going through the system, and the number of trials can vary from one to 400. Let's say that 200 trials are selected. Then, 200 trials of the simulation, each of 100 customers, will be conducted.

For Example 2.1, the frequency of waiting time in queue for the first trial of 100 customers is shown in Figure 2.7. (Note: In all histograms in the remainder of this chapter, the upper limit of the bin is indicated on the legend on the x-axis, even if the legend is shown centered within the bin.) As mentioned previously, 46% of them did not have to wait, and 42% waited less than four minutes (but more than zero minutes).

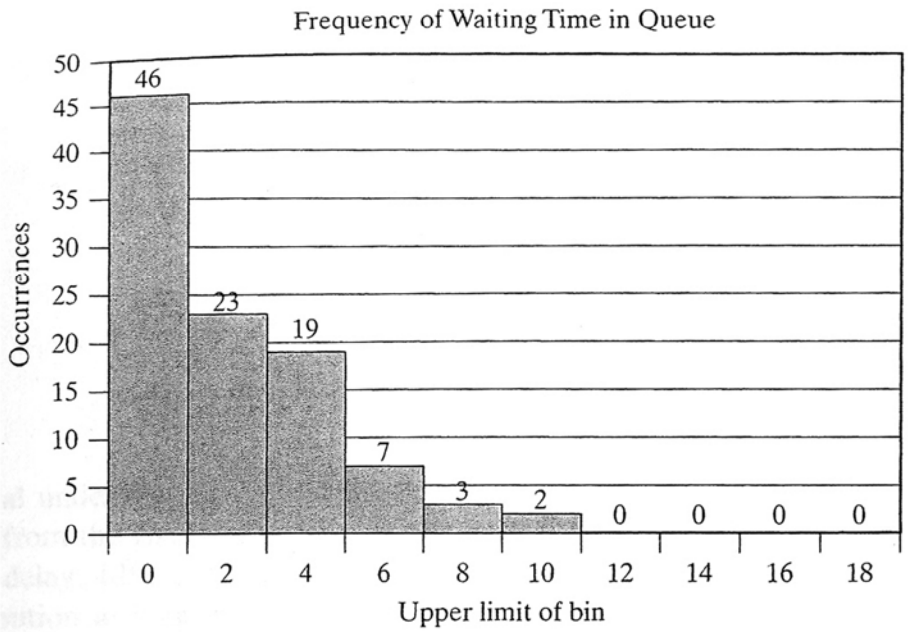

Figure 2.7 Frequency of waiting time in queue.

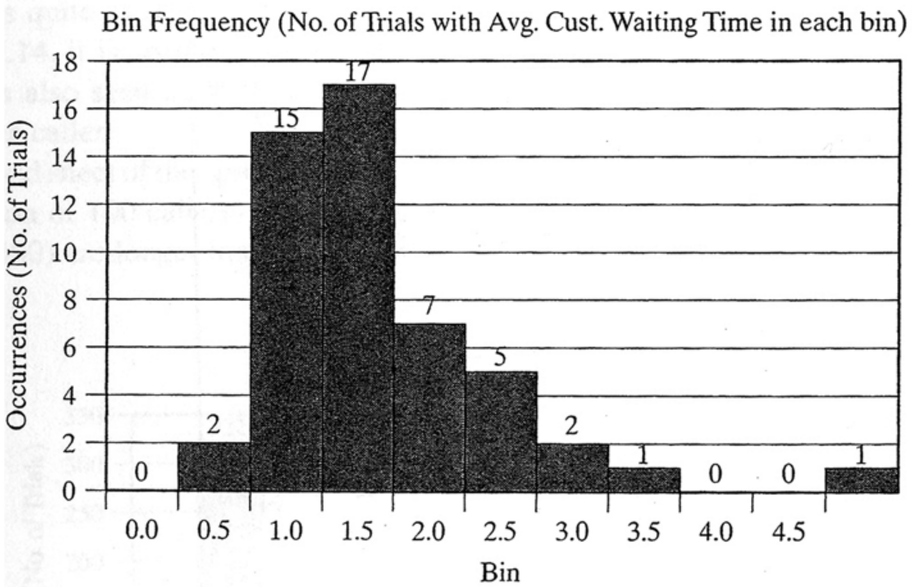

Figure 2.8 Frequency distribution of average waiting times.

From the Experiment sheet of the Excel spreadsheet, the average waiting time over 50 trials was 1.50 minutes. Figure 2.8 shows a histogram of the 50 average waiting times for the 50 trials. The overall average (1.50 minutes) is just to the right of the two most popular bins.

The exercises ask that you experiment with this spreadsheet. But, you can also experiment on your own to discover the effect of randomness and of the input data. For example, what if you run 400 trials instead of 50? Does the shape of the distribution in Figure 2.8 change? What if you run 25 trials instead of 50? How much does the shape change as you generate new trials?